EE105 – Fall 2014
Microelectronic Devices and Circuits

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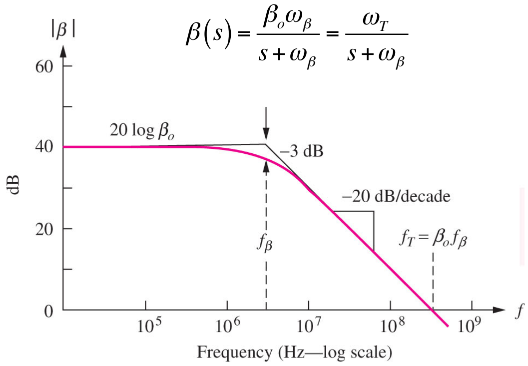
BJT Unity-Gain and Beta-Cutoff Frequencies

- **The current gain of the transistor decreases as the frequency increases and can be modeled by a single-pole transfer function.**

$$\beta(s) = \frac{\beta_o \omega_\beta}{s+\omega_\beta} = \frac{\omega_T}{s+\omega_\beta}$$



The "beta-cutoff frequency" of the BJT is called f_β :

$$\left|\beta(j\omega_\beta)\right| = \frac{\beta_o}{\sqrt{2}} \quad \text{(-3 dB pt.)}$$

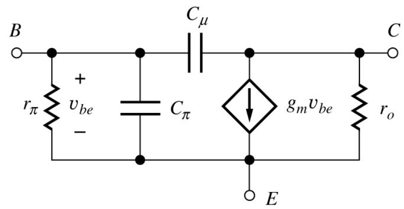
The "unity-gain frequency" is referred to as f_T :

$$\left|\beta(j\omega_T)\right| = 1 \quad \text{(0 dB)}$$

The two frequencies are related:

$$\omega_T = \beta_o \omega_\beta$$

Frequency-Dependent Transistor Models
Hybrid-Pi Model for the BJT



The frequency dependence of the BJT in forward-active region can be modeled by adding capacitors C_μ and C_π to the hybrid-pi model.

C_π models the change in base minority carrier charge as the base-emitter voltage of the transistor changes:

$$C_\pi = g_m \tau_F$$

where τ_F is the forward base transit-time of the transistor, the time a carrier takes to cross the base region. For BJT,

$$\tau_F = \frac{Q}{i_T} = \frac{W_B^2}{2D_n}, \quad \text{where}$$

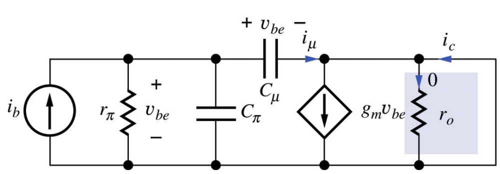
W_B is the base width,

D_n is the diffusion coefficient

C_μ is the capacitance of the reverse-biased collector-base diode:

$$C_\mu = \frac{C_{\mu o}}{\sqrt{1 + \left(V_{CB}/\phi_{jc}\right)}}$$

Beta-cutoff Frequency (ω_β) of BJT



We calculate the cutoff frequency for the short-circuit current gain using the circuit model given here.

$$\beta(s) = I_c(s)/I_b(s)$$

$$I_c(s) = \left(g_m - sC_\mu\right)V_{be}(s)$$

$$V_{be}(s) = I_b(s)\frac{r_\pi}{s\left(C_\pi + C_\mu\right)r_\pi + 1}$$

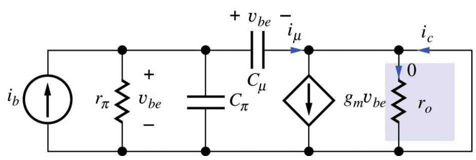
$$\beta(s) = \beta_o \frac{1 - sC_\mu/g_m}{s\left(C_\pi + C_\mu\right)r_\pi + 1}$$

The right-half plane transmission zero $\omega_Z = + g_m/C_\mu$ occurring at high frequency (above ω_T) can be neglected.

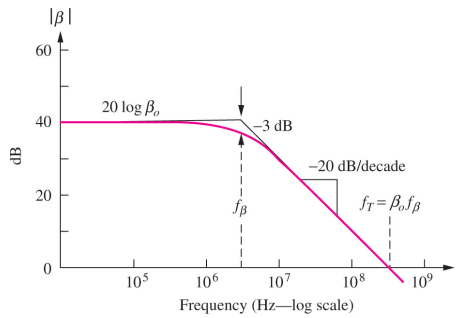
$$\therefore \beta(s) \cong \frac{\beta_o}{s\left(C_\pi + C_\mu\right)r_\pi + 1} = \frac{\beta_o}{\left(s/\omega_\beta\right) + 1}$$

where $\omega_\beta = 1/\, r_\pi(C_\mu + C_\pi)$ is the beta-cutoff frequency

Unity-gain Frequency (ω_T) of BJT



We can rearrange the current gain expression to expose the unity-gain frequency ω_T of the transistor.



$$\beta(s) \cong \frac{\beta_o}{(s/\omega_\beta)+1} = \frac{\beta_o \omega_\beta}{s+\omega_\beta} = \frac{\omega_T}{s+\omega_\beta}$$

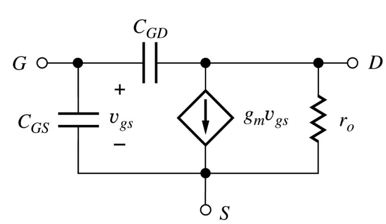
$$\omega_\beta = \frac{1}{r_\pi (C_\pi + C_\mu)}$$

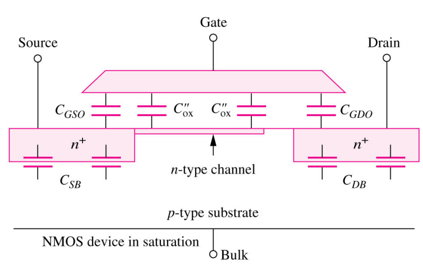
$$\omega_T = \beta_o \omega_\beta = \frac{g_m}{C_\pi + C_\mu}$$

$$f_T = \frac{\omega_T}{2\pi}$$

High-frequency Model for the MOSFET

At frequencies above dc, the input resistance and current gain of the MOSFET is no longer infinite. The pi-model for the MOSFET includes the gate-source and gate-drain capacitors C_{GS} and C_{GD}.

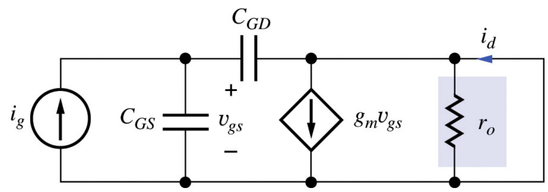




$$C_{GS} = \frac{2}{3} C_{GC} + C_{GSO} W \qquad C_{GD} = C_{GDO} W \qquad \text{where} \qquad C_{GC} = C''_{OX} WL$$

Unity-gain Frequency for the MOSFET

The short-circuit current gain and unity-gain frequency of the MOSFET are calculated in a manner similar to the BJT.



$$\beta(s) = \frac{I_d(s)}{I_g(s)}$$

$$\beta(s) = \frac{g_m}{s(C_{GS}+C_{GD})}\left(1 - s\frac{C_{gd}}{g_m}\right)$$

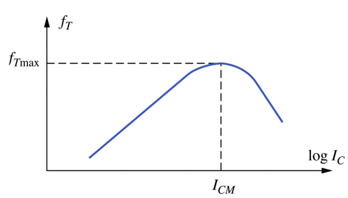
$$\beta(s) \cong \frac{g_m}{s(C_{GS}+C_{GD})} = \frac{\omega_T}{s}$$

$$\omega_T = \frac{g_m}{C_{GS}+C_{GD}}$$

$$\omega_T \cong \frac{\mu_n C_{ox}'' \frac{W}{L}(V_{GS}-V_{TN})}{(2/3)C_{ox}'' WL} = \frac{3}{2}\mu_n \frac{(V_{GS}-V_{TN})}{L^2}$$

Limitations of High-frequency Models

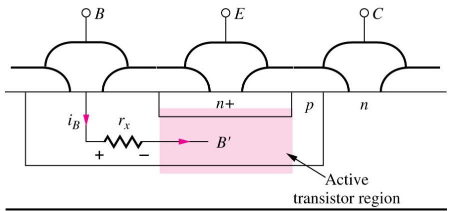
- **Above 0.3 f_T, behavior of simple pi-models begins to deviate significantly from the actual device.**
- **Also, ω_T depends on operating current as shown below and is not constant as assumed in the earlier slides.**
- **For given BJT, a collector current I_{CM} exists that yields f_{Tmax}.**

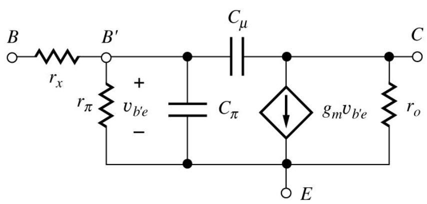


- **For the FET in saturation, C_{GS} and C_{GD} are independent of Q-point current, so**

$$\omega_T \propto g_m \propto \sqrt{I_D}$$

Bipolar Transistor Model
Base Resistance r_x





Base current enters the BJT through the external base contact and traverses a high resistance region before entering active area. Resistor r_x models the voltage drop between the base contact and the active area of the BJT.